Evaluating Japanese Foreign Exchange Intervention: a Simultaneous Equation Tobit Approach *

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Abstract

This study examines the efficacy of Japan's large-scale currency interventions between January 2003 and March 2004. Based on FIML estimator with ATE in a simultaneous equations Tobit model, we analyzed why interventions can be effective by considering their effects on exchange rates as their influence of an average trend. We considered these effects from two aspects based on structural estimation and used trend analysis for the estimation period based on a formulation of policy evaluation. Our empirical results indicated that buying dollars equivalent to one trillion yen could induce a same-day depreciation exceeding 1% in the yen. This effect is two-three times that resulting from usual regression analysis. Compared with the counterfactual case in which no intervention is assumed to have occurred on the day of intervention, our results suggest that the intervention effect was to nearly completely offset the appreciation trend in the yen.

Keywords: Foreign exchange intervention, Simultaneous equation Tobit model, FIML estimator, Test for endogeneity, Average treatment effect
JEL Classification: F31, C34, C54

1 Introduction

Numerous studies have investigated the effects of foreign exchange intervention employing various methods and perspectives. Exchange rate fluctuations occur because of intervention, or authorities intervene because of unexpected exchange rate fluctuations. Therefore, it is important to distinguish between the two interactions and to consider a simultaneous equation model. This paper examines the endogeneity of exchange rates and intervention, which might cause a problem in estimating the efficacy of intervention. In addition, we derive the effect of intervention by positing a counterfactual situation to

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measure its efficacy in the framework of program/policy evaluation developed in recent years. Nonetheless, endogeneity still cannot be ignored in this framework.

Among representative papers that examine simultaneity between exchange rate returns and intervention, Kearns and Rigobon (2005), Chen et al. (2012), and Iwata and Wu (2012) addressed the importance of endogeneity and derived the effect of intervention using Japanese data. Their models stem from Amemiya's (1974) simultaneous equation Tobit model. This model is especially apt for this purpose because intervention is an endogenous variable and it expresses the market response function regardless of whether intervention occurs or not. Although it is a structural estimation, we can utilize its feature that the market response function is always identified without exclusion conditions (cf. Maddalla, (1983)), whereas previous studies discuss the identification problems. On the other hand, it observes whether equilibrium exists (cf. Amemiya, (1974)).

Although those studies have established the effectiveness of Japanese intervention, this paper differs in three respects. First, it targets the great intervention in 2003 and 2004. Previous research did not examine this period, which is ideal for assessing intervention by establishing a counterfactual situation in the trend of the yen's appreciation despite active and extensive intervention. Second, we demonstrate how the volume of each day's intervention affects the exchange rate and examine the parameter of interest using a simple model of order flow following Lyons (2006). Third, we use a full information maximum likelihood (FIML) estimator, which is the efficient estimator for co-estimating market and policy response functions. Kearns and Rigobon (2005) conducted a simulated generalized method of moments (GMM) estimation, whereas Chen et al. (2012) conducted a Bayesian method (MCMC). Although the sample periods differ, our results exhibit effects similar to Kearns and Rigobon (2005) and Chen et al. (2012), which are larger than those under usual ordinary least squares (OLS) estimation.

In addition, we address three important econometric issues as extensions to previous studies. First, we propose test statistics for endogeneity since, to our knowledge, previous studies have not verified whether it exists. Second, we derive the average treatment effect (ATE) in a simultaneous equations Tobit model from its reduced forms. Fatum and Hutchison (2010) conducted one of the few studies that assess the efficacy of intervention in the framework of policy evaluation. This framework can clarify the policy effect using non-intervention as a benchmark. Compared with the ATE that Fatum and Hutchison (2010) conducted, our measurement of the ATE on the treated (ATE_1) is more suitable since ATE might hamper the efficacy of intervention. In this sense, we improve their results. Third, we propose that exchange rate volatility, usually estimated by a generalized autoregressive heteroskedasticity (GARCH) model, can be measured by a policy evaluation model. We demonstrate how today's intervention affects volatility. GARCH analysis does not measure the influence because it can be conditioned only on the previous day's intervention. Therefore, volatility is unchanged whether today's intervention occurs or not.

The paper is organized as follows. Section 2 reviews previous literature and describes

Japan's 2003-2004 foreign exchange intervention. Section 3 explains the parameters of interest and defines the intervention effect based on order flows. Section 4 summarizes the statistical estimation. Details appear in the Appendix. Section 5 provides empirical results. Section 6 concludes the paper.

2 Characteristics of the large foreign exchange intervention of 2003-2004

Japan's Ministry of Finance began publishing data about foreign exchange market intervention volume in August 2000. Figure 1 presents the data (monthly totals for January 1992-March 2004). Frequency and volume of intervention differ depending not only on changes in the exchange rates but also the policy set by the Vice Minister of Finance for International Affairs (Ito, (2003)). Intervention between January 2003 and March 2004 by Vice Minister Mizoguchi was unprecedented in size and frequency. Ito (2003) and Fatum and Yamamoto (2012) found that intervention volume significantly influenced exchange rates.

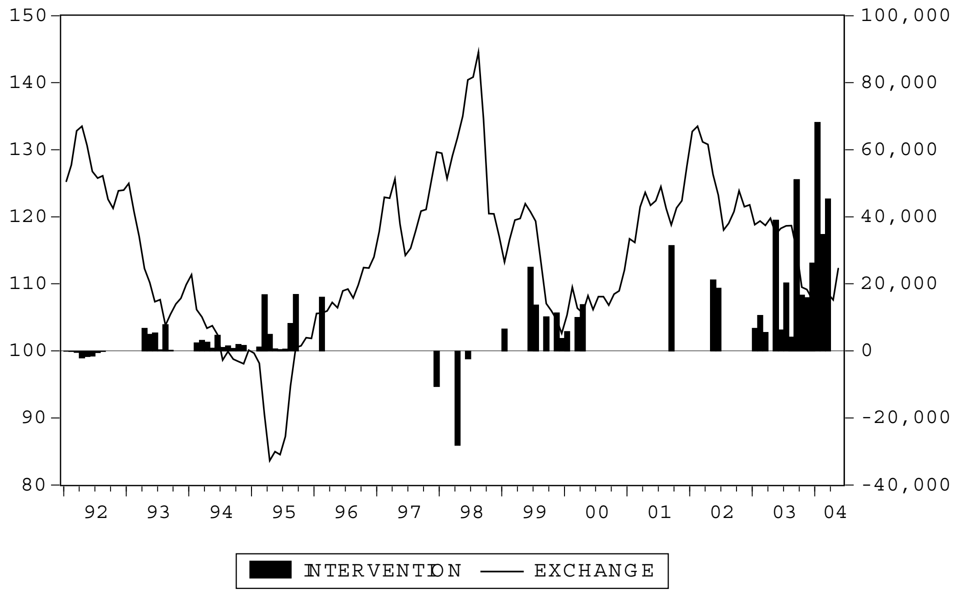

Fig. 1: Japanese intervention (100 million yen) and yen-dollar exchange rate (Monthly)

Figure 2 compares daily data of intervention against changes in exchange rates during the period overseen by Vice Minister Mizoguchi (January 2003-March 2004). It indicates that the Mizoguchi-Taylor Intervention from late 2003 to March 2004—named after Mizoguchi and John B. Taylor, the US Treasury's Under Secretary for International

Affairs—exceeded 1 trillion yen per day almost continuously throughout the period. This was apparently intended to counter sales of dollars acquired through Japan's trade surplus and extraordinary purchases of yen by foreign investment funds. The situation in Iraq surrounding September 2003 generated excessive capital inflows.

Despite active foreign exchange intervention during the period, however, the yen-dollar rate remained unchanged until February 2004. In particular, at the September 20, 2003, G7 meeting, media reports criticizing Japan's frequent intervention were followed by a sudden and dramatic appreciation of the yen. Therefore, it is difficult to obtain significant results when testing whether the period's large-scale intervention affected exchange rates. Fatum and Hutchinson (2010) estimated the ATE of their policy evaluation model and found that intervention from January 14, 2003-March 31, 2004 did not significantly affect rates. Ito (2004) compared this period with the period June 21, 1995-January 13, 2003 and found that the effect of intervention on exchange rates had decreased by half.

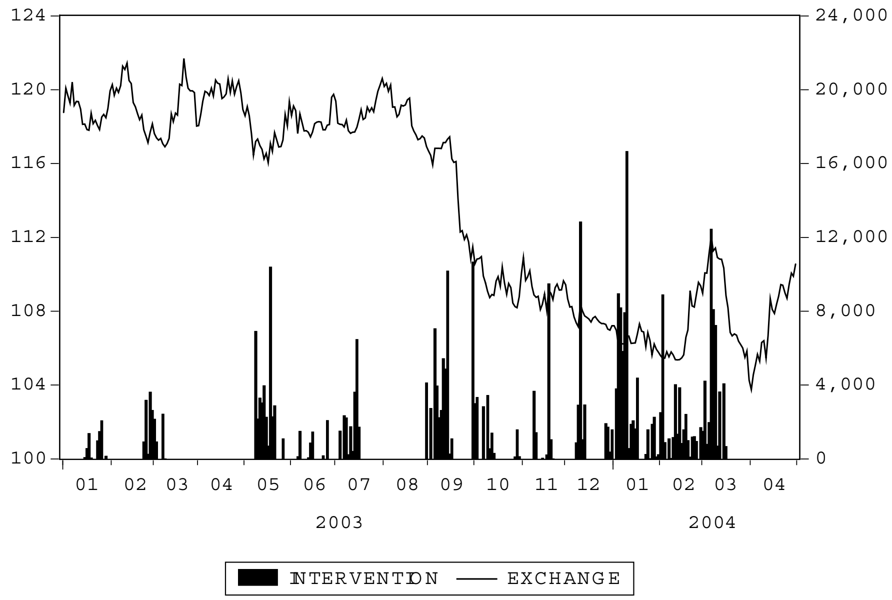

Fig. 2: Japanese intervention (100 million yen) and yen-dollar exchange rate (Daily)

Previous studies have been unable to estimate the effects of the period's massive intervention, but endogeneity substantially affects coefficients and ATE estimates. Reexamination is warranted. The next section discusses measuring the effect of intervention using order flows.

3 Efficacy of intervention policies

This section establishes the parameters of interest. The effects of foreign exchange intervention, as indicated by order flow, pass through to other market participants through individual trading. With reference to Lyons (2006, Ch. 7), the process leading up to interventions being reflected in prices can be explained as follows:

$$P_t = \psi_1 \sum_{\tau=1}^{t} \Delta R_\tau + \psi_2 \sum_{\tau=1}^{t} X_\tau \ , \tag{3.1}$$

$$\Delta P_t = \psi_1 \Delta R_t + \psi_2 X_t \ , \tag{3.2}$$

where P_t is exchage rate level, $\Delta P_t = P_t - P_{t-1}$ and ΔR_τ represents publicly observable macroeconomic information, as we assume that $\Delta R_\tau \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_R^2)$. Daily order flow X_t can be expressed as the grand total of the net transactions of n interbank dealers as follows:

$$X_t = \phi_1 \sum_{i=1}^{n} C_{it} = \phi_1 C_t \ , \tag{3.3}$$

where the customer's order $C_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_c^2)$. As equation (3.1) shows, P_t is eventually expressd as a random walk.

We slightly expand Lyons's framework (2006, Ch. 8) for measuring the efficacy of interventions. In addition to customer transactions, order flow includes intervention by a central bank. When a central bank intervenes in foreign exchange markets, interbank dealers can be categorized as bank i, which did not receive the order from the central bank, and bank j, which did. It follows that order flow X_t in the overall market is

$$\begin{aligned} X_t &= \sum_i \phi_1 C_{it} + \sum_j \phi_1 (C_{jt} + I_{jt}) \\ &= \phi_1(\phi_2 I_t + v_t) + \phi_1 I_t \ , \end{aligned} \tag{3.4}$$

where $I_t = \sum_j I_{jt}$ is the total amount of intervention. The second equality is based on empirical results in Girardin and Lyons (2008) or C_t is always written as

$$\begin{aligned} C_t &= \phi_2 I_t + (C_t - \phi_2 I_t) \\ &= \phi_2 I_t + v_t \ , \end{aligned} \tag{3.5}$$

where v_t is unobserved and possibly correlated with I_t . Thus, ϕ_2 is the indirect effect of intervention on hedge funds and similar entities. Then the change in exchange rates is

$$\Delta P_t = \beta_2 I_t + \psi_1 \Delta R_t + u_t \ , \tag{3.6}$$

where $u_t = \psi_2 \phi_1 v_t$ is treated as the error term. The I_t coefficient $\beta_2 = \psi_2(\phi_1\phi_2 + \phi_1)$ is the total effect, i.e., the sum of direct and indirect effects. The central bank is merely one customer that generates order flows; however, in published data, relatively large orders appear in connection with the central bank. Thus, the central bank sometimes is a unique customer able to influence order flows of other customers.

Let us consider effects of intervention in more detail. Because expectations for C_{it} and I_{jt} generally are not zero, considering $v_t \sim \mathcal{N}(\mu_c, \sigma_c^2)$ and $I_t \sim \mathcal{TN}(\mu_I, \sigma_I^2)$, if the intervention is always positive ($I_t > 0$), the distribution is truncated normal. In this case, P_t is a random walk with drift.

$$
\begin{aligned}
P_t &= \Big[\phi_1\psi_2\{(\phi_2\mu_I + \mu_c) + \mu_I\}\Big]t + \psi_1 \sum_{\tau=1}^{t} \Delta R_\tau + \psi_2 \sum_{\tau=1}^{t} X_\tau \ , \qquad (3.7) \\
\mathcal{E}[\Delta P_t] &= \phi_1\psi_2\{(\phi_2\mu_I + \mu_c) + \mu_I\} \\
&= \beta_2\mu_I + \psi_2\phi_1\mu_c \ . \qquad (3.8)
\end{aligned}
$$

In this case, the effect of intervention can be interpreted by its influence on the deterministic linear trend $\mathcal{E}[\Delta P_t]t$. Since there are days with and without intervention in practice, we must consider a Tobit model or $0 < \Pr(I_t = 0) = 1 - p < 1$. If we assume I_t is independent of u_t, then

$$
\begin{aligned}
\mathcal{E}[\Delta P_t] &= (\beta_2\mu_I + \psi_2\phi_1\mu_c)p + (\psi_2\phi_1\mu_c)(1-p) \\
&= \alpha_1 p + \alpha_0(1-p) \ , \qquad (3.9)
\end{aligned}
$$

we attempt to measure changes in the trend with ATE or $\alpha = \alpha_1 - \alpha_0 = \beta_2\mu_I$.

The marginal effect at t is given by β_2. If selling causes the yen to depreciate, β_2 is negative. We define the efficacy of intervention in some period ($t = 1, ..., T$) as follows:

$$
\alpha < 0 \ , \quad \alpha_1 < 0 \ . \qquad (3.10)
$$

If $\beta_2\mu_I < -\psi_2\phi_1\mu_c$ or $\alpha_1 < 0$, intervention launches a trend $\alpha_1 t$ of depreciating exchange rates, then intervention is strongly effective.

When estimating β_2 and α, the ultimate issue is endogeneity of the intervention. In other words, the amount of intervention I_t and the error term u_t in equation (3.6) are likely correlated.

$$
\mathcal{C}or[I_t,\ u_t] \neq 0 \ . \qquad (3.11)
$$

In this case, the regression analysis contains bias.

Endogeneity occurs for several reasons. If we assume ϕ_2 in equation (3.5) approaches 0, then $u_t \propto C_t$. However, other order flows C_t and intervention I_t conceivably are correlated (Girardin and Lyons, (2008)), causing omitted variable bias because C_t is unobserved. In their examination of intra-day trading, Chen et al. (2012) demonstrated the presence of endogeneity by aggregating hourly data, and OLS estimator suffered aggregation bias. Ito (2003) doubted whether I_t is an exogenous variable. Japan's intervention in the 2003-2004 period was large and continuous. Authorities presumably observed movements in exchange rates the day before intervening, and their response function incorporated whether and to what extent they should intervene. This situation yields two simultaneous equations and generates simultaneous equation bias.

Although we cannot measure the effects of order flow because we lack data for C_t, this section demonstrated what is the parameter of interest. The next section considers endogeneity in standard simultaneous equation models and verifies whether it is present.

4 Statistical inference of simultaneous equation Tobit model

4.1 The FIML estimator and test statistics for endogeneity

This section presents two structural equations for periods $(t = 1, ..., T)$ adopting econometric notations.

$$y_t^{(1)*} = \beta_1 y_t^{(2)} + \boldsymbol{\gamma}_1' \mathbf{z}_t^{(1)} + u_t^{(1)} \ , \tag{4.12}$$

$$y_t^{(1)} = \begin{cases} y_t^{(1)*} & \text{if} \quad y_t^{(1)*} > 0 \ , \\ 0 & \text{if} \quad y_t^{(1)*} \leq 0 \ . \end{cases} \tag{4.13}$$

$$y_t^{(2)} = \beta_2 y_t^{(1)} + \boldsymbol{\gamma}_2' \mathbf{z}_t^{(2)} + u_t^{(2)} \ , \tag{4.14}$$

where there are two endogenous variables $y_t^{(1)} = I_t$, which is the yen-selling (dollar-buying) volume, and $y_t^{(2)} = \Delta P_t$, which is the change in the dollar-yen exchange rate. $(\mathbf{z}_t^{(1)}, \mathbf{z}_t^{(2)})$ denotes instrumental variable vectors. For example, the constant, which is the drift term, and the difference between US and Japanese interest rates $\Delta R_t = \Delta(i_t^* - i_t)$ are considered exogenous variables. Therefore, $(\mathbf{z}_t^{(1)}, \mathbf{z}_t^{(2)})$ are subsets of $\mathbf{z}_t = (\ 1,\ \Delta R_t,\ I_{t-1},\ \Delta P_{t-1},\ I_{t-2},\ \Delta P_{t-2}, ...)$, which includes the exogenous and predetermined variables. Thus, we have extended the model because coefficients of lagged dependent variables might not be zero.

The first structural equations—(4.12) and (4.13)—are the authorities' response functions. Intervention allows the possibility of responding with exchange rate variation $y_t^{(2)}$ in the current day. Positive intervention is observed only when latent variable $y_t^{(1)*}$ exceeds the threshold. However, during the examined period, intervention involved only selling yen; therefore, this formulation holds. The second structural equation (4.14) is the response function of foreign exchange markets, which is not influenced on days $(y_t^{(1)} = 0)$ during which no intervention occurred. Figure 3 shows relationships of the endogenous variables when instrumental variables are givens. β_2 corresponds to the total intervention effect. Noting that this is the dollar-yen exchange rate, authorities sell more yen when it appreciates, and the expected sign of β_1 is positive. Conversely, the expected sign of β_2 is negative when selling causes the yen to depreciate.

The equilibrium point or observable reduced form in Figure 3 exists if condition $(1 - \beta_1\beta_2) > 0$ holds as given by the following (Nelson and Olson, (1978)).

$$y_t^{(1)} = (\boldsymbol{\pi}_1' \mathbf{z}_t + v_t^{(1)}) w_t \ , \tag{4.15}$$

$$y_t^{(2)} = (\boldsymbol{\pi}_2' \mathbf{z}_t + v_t^{(2)}) w_t + (\boldsymbol{\gamma}_2' \mathbf{z}_t^{(2)} + u_t^{(2)})(1 - w_t) \ , \tag{4.16}$$

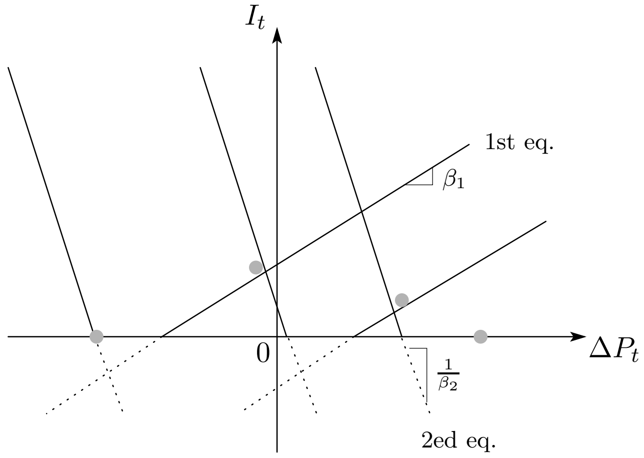

Fig. 3: Simultaneous equation Tobit model

where, if we use the selection matrix $\mathbf{J}^{(1)}$ and $\mathbf{J}^{(2)}$ and express instrumental variable vectors as $\mathbf{z}_t^{(1)} = \mathbf{J}^{(1)}\mathbf{z}_t$ and $\mathbf{z}_t^{(2)} = \mathbf{J}^{(2)}\mathbf{z}_t$, the reduced coefficients and error terms $\mathbf{v}_t = (v_t^{(1)}, v_t^{(2)})'$ are given by

$$\mathbf{B} = \begin{bmatrix} 1 & -\beta_1 \\ -\beta_2 & 1 \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{\pi}_1' \\ \boldsymbol{\pi}_2' \end{bmatrix} = \mathbf{B}^{-1} \begin{bmatrix} \boldsymbol{\gamma}_1'\mathbf{J}^{(1)} \\ \boldsymbol{\gamma}_2'\mathbf{J}^{(2)} \end{bmatrix}, \quad \mathbf{v}_t = \mathbf{B}^{-1} \begin{bmatrix} u_t^{(1)} \\ u_t^{(2)} \end{bmatrix}. \tag{4.17}$$

In addition, w_t is $w_t = \mathbb{1}\{y_t^{(1)*} > 0\} = \mathbb{1}\{\boldsymbol{\pi}_1'\mathbf{z}_t + v_t^{(1)} > 0\}$, an indicator function whose value is 1 if the argument is true and 0 otherwise. Depending on whether intervention occurs ($w_t = 1$) or not ($w_t = 0$), the reduced form becomes a switching regression model. This is a special form of the simultaneous equation Tobit model proposed by Amemiya (1974). Amemiya (1974) discovered that as it has simultaneous inequalities, the precondition for existence of the equilibrium is $|\mathbf{B}| = (1 - \beta_1\beta_2) > 0$, which is termed the coherence condition. This condition is met as indicated by the expected signs.

Intervention does not occur daily (Figure 2). In other words, the endogenous variable $y_t^{(1)}$ is censored at 0 in this Tobit model, and selection bias arises in the OLS estimation. The is also a simultaneous equation model. Even if we analyze the equilibrium point in Figure 3, its regression line is neither the first nor the second structural equation.

The FIML estimator derived from the simultaneous estimation of the two structural equations is defined by Amemiya (1974) and is the efficient estimator. Denoting the $(y_t^{(1)*}, y_t^{(2)})$ density function as f and the $(u_t^{(1)}, u_t^{(2)})$ density function as g, where the bivariate normal distribution follows that $(u_t^{(1)}, u_t^{(2)})' \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \boldsymbol{\Sigma} = (\sigma_1^2, \sigma_{12}; \sigma_{12}, \sigma_2^2)$.

Then the likelihood function for period t is given by

$$
\begin{aligned}
l_t &= \left[\, f(y_t^{(1)}, y_t^{(2)})\, \right]^{w_t} \left[\, \int_{-\infty}^{0} f(y^{(1)*},\ y_t^{(2)}) dy^{(1)*}\, \right]^{(1-w_t)} \\
&= \left[\, (1-\beta_1\beta_2)\, g(y_t^{(1)} - \beta_1 y_t^{(2)} - \boldsymbol{\gamma}_1' \mathbf{z}_t^{(1)},\ y_t^{(2)} - \beta_2 y_t^{(1)} - \boldsymbol{\gamma}_2' \mathbf{z}_t^{(2)})\, \right]^{w_t} \\
&\quad \times \left[\, \int_{-\infty}^{-\beta_1 y_t^{(2)} - \boldsymbol{\gamma}_1' \mathbf{z}_t^{(1)}} g(u^{(1)},\ y_t^{(2)} - \boldsymbol{\gamma}_2' \mathbf{z}_t^{(2)}) du^{(1)}\, \right]^{(1-w_t)} . \qquad (4.18)
\end{aligned}
$$

According to Maddala (1983, Ch. 7), the second structural equation (4.14) is always identified. However, the first structural equation (4.12) requires the usual exclusion conditions in linear simultaneous equation models. For instrumental variables absent in the first structural equation, $\mathcal{Cor}[I_t, \Delta R_t] = 0$ if intervention is theoretically sterilized. Therefore, we considered an exclusion condition, such as differences in Japanese and US interest rates. The likelihood function for the FIML estimation is represented as an integral, which needs adjustment to maximize the function. We used a representation involving only a univariate standard normal distribution with orthogonal transformation. The Appendix provides the specific likelihood function and its derivation.

We assumed intervention volume and exchange rates are determined simultaneously by equations (4.12) and (4.14). In this case, $y_t^{(1)}$ and $y_t^{(2)}$ generally correlate with the error terms in the structural equations. However, parameter values may weaken the correlation and require testing. For instance, if $y_t^{(1)*}$ could be observed, a triangular system ($\beta_1 = 0$) and $\sigma_{12} = 0$, then we have $\mathcal{Cor}[y_t^{(1)*},\ u_t^{(2)}] = 0$ with no endogeneity. In such cases, equation (4.12) can be consistently estimated with a standard Tobit estimation or equation (4.14) via OLS. The null hypothesis asserts that no endogeneity exists. In the first structural equation, it is denoted as $\mathrm{H}_0^{(1)} : \mathcal{E}[y_t^{(2)} u_t^{(1)}] = 0$, and for the second structural equation it is denoted as $\mathrm{H}_0^{(2)} : \mathcal{E}[y_t^{(1)} u_t^{(2)}] = 0$. The specification test for endogeneity employs a residual regression based on Hausman (1978) or Wooldridge (2006, Ch. 6). However, it cannot be applied to our reduced form in which the endogenous variable is censored. We thus propose these test statistics as sample analogues for the null hypotheses:

$$
z_{(1)} = \frac{1}{\sqrt{T}\hat{\sigma}_{(1)}} \sum_{t=1}^{T} (\hat{\beta}_2 y_t^{(1)} \hat{u}_t^{(1)} + \hat{\sigma}_{12}) \xrightarrow{d} \mathcal{N}(0,1)\ , \qquad (4.19)
$$

$$
z_{(2)} = \frac{1}{\sqrt{T}\hat{\sigma}_{(2)}} \sum_{t=1}^{T} y_t^{(1)} \hat{u}_t^{(2)} \xrightarrow{d} \mathcal{N}(0,1)\ . \qquad (4.20)
$$

See the Appendix for the derivation. Under the null hypotheses, these z statistics follow a standard normal distribution. We use a two-sided test.

4.2 ATE and VTE of simultaneous equation Tobit model

Recent studies, including Fatum and Hutchinson's (2010), measure ATE in models evaluating intervention policies. Our model considers two issues not commonly addressed.

In a time series analysis, the causal effect of a policy intervention is defined as

$$y_t^{(2)} = y_{1t}^{(2)} w_t + y_{0t}^{(2)} (1 - w_t) \ , \tag{4.21}$$

where $y_{1t}^{(2)}$ and $y_{0t}^{(2)}$ are policy responses when intervention occurred and when it did not, respectively. One can be observed; the other is counterfactual. For more general formulation, see Jordà and Taylor (2013) and Hsiao et al. (2012). We first note that the reduced form (4.16) has the same value as equation (4.21). In other words, it is given by

$$y_{1t}^{(2)} = \boldsymbol{\pi}_2' \mathbf{z}_t + v_t^{(2)} \ , \quad y_{0t}^{(2)} = \boldsymbol{\gamma}_2' \mathbf{z}_t^{(2)} + u_t^{(2)} \ . \tag{4.22}$$

The first issue is the problem that arises in estimating ATE: policy $w_t = \mathbb{I}\{\boldsymbol{\pi}_1' \mathbf{z}_t + v_t^{(1)} > 0\}$ is also an endogenous variable. Rosenbaum and Rubin's (1983) general assumption of conditional independence underlies approaches such as propensity score matching. $(y_{1t}^{(2)}, y_{0t}^{(2)}, w_t)$ also depends on error terms $(v_t^{(1)}, v_t^{(2)})$, so this assumption is not satisfied. However, ATE is easily estimated using the reduced form since it is solved by instrumental variables and error terms.

$$\begin{aligned} \text{ATE} &= \mathcal{E}\left[y_{1t}^{(2)} - y_{0t}^{(2)}\right] \\ &= \mathcal{E}\left[y_{1t}^{(2)}\right] - \mathcal{E}\left[y_{0t}^{(2)}\right] \\ &= \alpha_1 - \alpha_0 \\ &= \boldsymbol{\pi}_2' \mathcal{E}\left[\mathbf{z}_t\right] - \boldsymbol{\gamma}_2' \mathcal{E}[\mathbf{z}_t^{(2)}] \ . \end{aligned} \tag{4.23}$$

Given the stationarity of $\mathbf{z}_t$, we can make a consistent estimator using the sample average $\bar{\mathbf{z}}_t = (1/T) \sum_{t=1}^{T} \mathbf{z}_t$.

Moreover, the model includes the latent variable $y_t^{(1)*} < 0$ in equation (4.13). The second issue is that ATE is not necessarily an appropriate assessment criterion. The Appendix derives the following useful alternative representation for ATE.

$$y_{1t}^{(2)} - y_{0t}^{(2)} = \beta_2 (\boldsymbol{\pi}_1' \mathbf{z}_t + v_t^{(1)}) \ , \tag{4.24}$$

$$\begin{aligned} \mathcal{E}\left[y_{1t}^{(2)} - y_{0t}^{(2)}\right] &= \beta_2 \mathcal{E}\left[\ y_t^{(1)*} \mathbb{I}\{y_t^{(1)*} > 0\} + \frac{1}{1 - \beta_1 \beta_2} y_t^{(1)*} \mathbb{I}\{y_t^{(1)*} \leq 0\}\ \right] \\ &= \beta_2 \mathcal{E}\left[\ y_t^{(1)} \mathbb{I}\{y_t^{(1)*} > 0\}\right] + \frac{\beta_2}{1 - \beta_1 \beta_2} \mathcal{E}\left[y_t^{(1)*} \mathbb{I}\{y_t^{(1)*} < 0\}\ \right] , \end{aligned} \tag{4.25}$$

where $y_t^{(1)*} < 0$ is a hypothetical yen-buying intervention. If $\beta_2 < 0$, either buying or selling yen would be effective, i.e., each term of (4.25) is not zero, but ATE cancels both out by virtue of its averaging. Therefore, ATE may lead to underestimating the efficacy of yen-selling. This is similar to the discussion of the monotonicity of LATE in Imbens and Angrist (1994). Hypothetical yen-buying intervention $y_t^{(1)*}$ in the negative region of Figure 1 was never realized during the examined period, and it was substituted by $y_t^{(1)}=0$ in the case of no intervention. Thus, ATE is unsuitable.

We must extract the effect of yen-selling where intervention volume is positive in (4.25). Following Wooldridge (2006, Ch. 18), we address this situation using the average treatment effect (ATE_1: ATE on the treated) when the policy is implemented. This approach has been emphasized in recent years. From equations (4.15) and (4.25), $\mathrm{ATE}_1=\alpha_{\cdot 1}$ is the following:

$$\begin{aligned}\mathrm{ATE}_1 &= \mathcal{E}\left[y_{1t}^{(2)}-y_{0t}^{(2)}\middle|w_t=1\right] \\ &= \alpha_{1\cdot 1}-\alpha_{0\cdot 1} \\ &= \beta_2\mathcal{E}\left[y_t^{(1)}\middle|w_t=1\right]\ . \end{aligned} \tag{4.26}$$

Consistent estimation is easily achieved with $\hat{\alpha}_{\cdot 1}=(\hat{\beta}_2\sum_t y_t^{(1)}w_t)/\sum_t w_t$. In addition, the exchange rate deterministic trend $E[y_{0t}^{(2)}|w_t=1]t=\alpha_{0\cdot 1}t$ in the case where no intervention occurred even on the day of intervention, which is of more interest, is expressed as follows:

$$\alpha_{0\cdot 1}=\mathcal{E}\left[\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)}+\frac{\sigma_{12}+\beta_1\sigma_2^2}{(1-\beta_1\beta_2)\omega_1}\frac{\phi(\boldsymbol{\pi}_1'\mathbf{z}_t)}{\Phi(\boldsymbol{\pi}_1'\mathbf{z}_t)}\middle|w_t=1\right]\ . \tag{4.27}$$

Although it is possible to estimate $\alpha_{0\cdot 1}$ from the above sample analogue, a simple method is available. $\alpha_{1\cdot 1}=\mathcal{E}[y_{1t}^{(2)}|w_t=1]$ can be considered as $\hat{\alpha}_{1\cdot 1}=\sum_t y_t^{(2)}w_t/\sum_t w_t$. Calculating backward from $\hat{\alpha}_{\cdot 1}$, we obtain

$$\hat{\alpha}_{0\cdot 1}=\hat{\alpha}_{1\cdot 1}-\hat{\alpha}_{\cdot 1}\ . \tag{4.28}$$

If $\alpha_{0\cdot 1}$ is positive, yen-selling will cancel the trend of appreciation. It can be interpreted as a policy of "leaning against the wind." If negative, policy of "leaning with the wind" augments the appreciation. The Appendix shows the derivation of the t-test statistic for the null hypothesis $\mathrm{H}_0:\alpha_{\cdot 1}=\alpha_{1\cdot 1}-\alpha_{0\cdot 1}=0$ using the delta method.

As our second concern, we validate the volatility of changes in exchange rates. Traditionally, this situation would suit GARCH estimation. Here, we proposes the following framework for evaluating policy.

$$\mathrm{VTE}=\mathcal{V}ar\left[y_{1t}^{(2)}\right]-\mathcal{V}ar\left[y_{0t}^{(2)}\right]\ . \tag{4.29}$$

In other words this measure compares the difference in variances due to the treatment. We term this VTE. Note that VTE differs from the variance of treatment effect $\mathcal{V}ar[y_{1t}^{(2)}-y_{0t}^{(2)}]$, i.e.,

$$\mathcal{V}ar\left[y_{1t}^{(2)}-y_{0t}^{(2)}\right]=\mathcal{V}ar\left[y_{1t}^{(2)}\right]-2\mathcal{C}ov\left[y_{1t}^{(2)},y_{0t}^{(2)}\right]+\mathcal{V}ar\left[y_{0t}^{(2)}\right]\ . \tag{4.30}$$

Similar to ATE_1, we propose VTE_1 (VTE on the treated) as follows:

$$\mathrm{VTE}_1=\sigma_{1\cdot 1}^2-\sigma_{0\cdot 1}^2\ , \tag{4.31}$$

where, $\sigma^2_{1\cdot 1} = \mathcal{V}ar[y^{(2)}_{1t}|w_t = 1]$ and $\sigma^2_{0\cdot 1} = \mathcal{V}ar[y^{(2)}_{0t}|w_t = 1]$. Hence, if VTE_1 is negative, variance would decline using the case of no intervention as a benchmark. From equations (4.24) and $\mathcal{V}ar[y^{(2)}_{1t}|w_t = 1] = \mathcal{V}ar[y^{(2)}_{t}|w_t = 1]$, we have the relation

$$\sigma^2_{1\cdot 1} = \beta_2^2 \mathcal{V}ar\left[y^{(1)}_t \middle| w_t = 1\right] + 2\beta_2 \mathcal{C}ov\left[y^{(1)}_t, y^{(2)}_{0t} \middle| w_t = 1\right] + \sigma^2_{0\cdot 1} \ , \tag{4.32}$$

where the first term is the volatility caused by intervention itself and the second term is the effect of a policy of "leaning against the wind." The expected sign of VTE_1 depends on whether the first or the second term of equation (4.32) is larger and is therefore not trivial. The VTE_1 estimator and t-test statistic for the null hypothesis $\mathrm{H}_0 : \sigma^2_{1\cdot 1} = \sigma^2_{0\cdot 1}$ are in the Appendix.

5 Empirical analysis

5.1 Model of analysis

For explanatory variables in this analysis, we consulted the studies by Ito (2003) and Evans and Lyons (2002). Specification of the two structural equations is as follows:

$$\begin{aligned} I^*_t &= \beta_1 \Delta P_t + \gamma_{10} + \gamma_{11}\Delta P_{t-1} + \gamma_{12}\Delta P_{t-2} + \gamma_{13} I_{t-1} \\ &\quad + \gamma_{14} I_{t-2} + \gamma_{15}\Delta P_{t-3:7} + \gamma_{16}\Delta P_{t-8:30} + u^{(1)}_t \ , \end{aligned} \tag{5.33}$$

$$I_t = \begin{cases} I^*_t & \text{if} \quad I^*_t > 0 \ , \\ 0 & \text{if} \quad I^*_t \leq 0 \ . \end{cases} \tag{5.34}$$

$$\begin{aligned} \Delta P_t &= \beta_2 I_t + \gamma_{20} + \gamma_{21}\Delta R_t + \gamma_{22}\Delta P_{t-1} + \gamma_{23}\Delta P_{t-2} \\ &\quad + \gamma_{24} I_{t-1} + \gamma_{25} I_{t-2} + \gamma_{26} I_{t-7} + \gamma_{27} I_{t-30} + u^{(2)}_t \ , \end{aligned} \tag{5.35}$$

where I_t is yen-selling/dollar-buying volume (trillion yen) at period t, and $\Delta P_t = (P_t - P_{t-1}) \times 100$ is the difference in the logarithm of the exchange rate (i.e., approximately the percentage change in the dollar-yen exchange rate on the New York Stock Exchange (NYSE)).

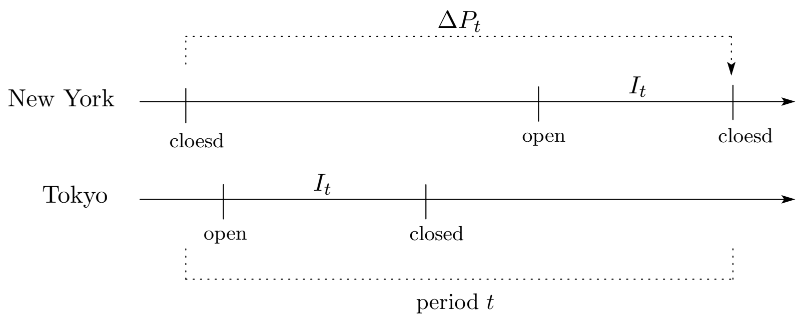

Fig. 4: Definition of period t

We are focused on the simultaneity of $y_t^{(1)} = I_t$ and $y_t^{(2)} = \Delta P_t$ during period t, so period t must be defined. Figure 4 defines it as the period spanning the end of trading on one day to the end of trading the next. Hence, intervention volume I_t is treated as the total amount of intervention during this period in New York, Tokyo, and elsewhere.

Other variables are instrumental variables. $\Delta R_t = \Delta(i_t^* - i_t)$ is the difference between US and Japanese interest rates (US O/N−Japanese O/N). Concerning the predetermined variables in the authorities' response function (5.33), $\Delta P_{t-s:u} = (P_{t-s} - P_{t-u}) \times 100$ represents the cumulative percentage change from three to seven days prior or from eight to thirty days prior. During numerical simulation, small interventions occurred almost daily but only with the first- and second-order lagged variables. This finding is inconsistent with Figure 2. Therefore, we included these variables $\Delta P_{t-s:u}$ under the assumption that authorities consider medium-term changes in exchange rates.

5.2 Results of estimation and tests

The estimation employs daily data from January 1, 2003, to April 28, 2004. There were 55 interventions during the earlier period and 74 during the latter. Exogenous factors strongly influenced exchange rate variations from September 19-22, 2003, so we omitted this period from analysis. The test statistic for structural changes was $q = 36.9$, and 31.4 at the 95% level of $\chi^2(20)$; therefore, the null hypothesis was rejected at 5% significance[1]. Hence, we divided the results of the FIML estimation $(\hat{\boldsymbol{\theta}}_{03}, \hat{\boldsymbol{\theta}}_{04})$ into the earlier and later periods (Table 1). The coherence conditions $(1 - \hat{\beta}_1\hat{\beta}_2) > 0$ were also satisfied, indicating that the simultaneous equation Tobit model holds. Tests for endogeneity revealed $z_{(1)} = -0.01$ and $z_{(2)} = 2.37$ for the earlier period and $z_{(1)} = 0.07$ and $z_{(2)} = 1.44$ for the later period. We conclude there is no endogeneity in the first structural equation, and the endogeneity exists in the second structural equation for the earlier period, thus necessitating FIML estimation.

First, the crucial result is the total effect β_2 of intervention in the second structural equation. The sign of the intra-day influence of foreign exchange intervention is negative, as expected. Further, it is the most significant of all coefficients for both periods, with the magnitude of the coefficients being maximum compared with the intervention effect leading up to the prior day. In light of the estimated coefficients, the effect of a one trillion yen intervention in the earlier period is a depreciation in the yen's value of about 1.6% (1.0% in the later period). As the exchange rate at the time was one dollar to approximately 100 yen, Figure 2 reveals that selling one trillion yen induced a depreciation exceeding one yen.

Regarding the influence of endogeneity, the sign of statistic $z_{(2)}$ allows us to predict an upward bias in the OLS estimation. In practice, estimating the second structural equation using OLS yields $\tilde{\beta}_2 = -0.52$ for the earlier period and $\tilde{\beta}_2 = -0.43$ for the later period. In contrast, the FIML estimated values are $\hat{\beta}_2 = -1.59$ and $\hat{\beta}_2 = -1.05$

[1] Derivation of the test statistic for structural changes is in the Appendix.

Table 1: FIML estimation results

	'03/1/1 ∼ '03/9/18		'03/9/23 ∼ '04/4/28	
	Dep. var.= I_t	Dep. var.= ΔP_t	Dep. var.= I_t	Dep. var.= ΔP_t
ΔP_t	0.21		0.16	
	(0.29)		(0.39)	
I_t		-1.59^{***}		-1.05^{***}
		(−3.69)		(−2.59)
const.	−0.49	0.10^{**}	-0.22^{*}	0.13^{*}
	(−1.02)	(2.06)	(−1.78)	(1.77)
ΔR_t		−0.14		0.48
		(−0.24)		(0.32)
ΔP_{t-1}	0.36	−0.06	0.17^{*}	0.07
	(1.00)	(−0.78)	(1.93)	(0.80)
ΔP_{t-2}	0.22	0.12	−0.04	-0.16^{*}
	(1.43)	(1.56)	(−0.47)	(−1.84)
I_{t-1}	0.83	0.38	0.67^{***}	0.56^{**}
	(1.36)	(1.37)	(4.10)	(2.45)
I_{t-2}	0.68^{*}	0.66^{**}	0.29	−0.06
	(1.84)	(2.34)	(1.57)	(−0.31)
$\Delta P_{t-3:7}$	0.16		−0.04	
	(1.22)		(−1.08)	
$\Delta P_{t-8:30}$	0.06		−0.01	
	(1.01)		(−0.45)	
I_{t-7}		-0.66^{**}		0.17
		(−2.36)		(1.14)
I_{t-30}		−0.18		-0.38^{**}
		(−0.58)		(−2.40)
σ_1, σ_2	0.36	0.49^{***}	0.45^{***}	0.55^{***}
	(1.45)	(15.5)	(3.42)	(13.1)
σ_{12}	0.05		0.05	
	(0.49)		(0.44)	
ATE_1	-0.385^{***}		-0.309^{**}	
	(−3.758)		(−2.283)	
$\hat{\alpha}_{1\cdot 1}$, $\hat{\alpha}_{0\cdot 1}$	0.002	0.387	0.018	0.327
VTE_1	−0.074		−0.116	
	(−0.381)		(−0.517)	
obs.	187		157	
$1-\hat{\beta}_1\hat{\beta}_2$	1.33		1.02	
$z_{(1)}$, $z_{(2)}$	−0.01	2.38	−0.50	1.35
$R^2_{(1)}$, $R^2_{(2)}$	0.42	0.08	0.25	0.06

1) t-values are shown in parentheses. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

2) The instrumental variables that do not appear in the first structural equation are ΔR_t, I_{t-7}, I_{t-30}.

for the earlier and later periods, respectively, which are two to three times greater than the OLS estimates. Chen et al. (2012) and Kearns and Rigobon (2005) examined the period up to 2002, estimating an intervention effect of approximately 1.7% and 1.5%, respectively, and demonstrating OLS estimation results more than two times greater due to the influence of endogeneity. Although time periods differ, the similarity in results is noteworthy.

Examining other variables in the second structural equation reveals that the difference in US-Japanese interest rates is not significant. This result might be an effect of zero interest rate policies. Based on the theory of price formation via order flow, only intervention volume on that day can have an impact. However, during this period, intervention volume one or two days prior shows positive significance. This can be interpreted as movement resulting from selling yen. For example, dealers who carried a sell yen/buy dollars position over to the next day may have been taking profits. In other words, foreign exchange intervention might not affect exchange rates uniformly. Subsequent profit-taking by dealers who create overnight positions in anticipation of intervention-induced order flow can produce market movements opposite to those produced by intervention.

The coefficients of determination $R^2_{(2)}$ are nearly 0[2]. Exchange rates cannot be predicted from percentage changes and interventions the previous day; they are almost completely explained by order flows during the period t. As concerns the trend or ATE_1 when intervention does occur, intervention on the period t had a large influence. As we have $\hat{\alpha}_{1\cdot 1} = 0.002(\%)$ in the earlier period and $\hat{\alpha}_{1\cdot 1} = 0.018(\%)$ in the later period, therefore intervention was not strongly effective ($\hat{\alpha}_{1\cdot 1} > 0$) but the trend in the yen's appreciation on intervention days is almost zero.

For example, in Figure 2, the daily average percentage change during 111 days between September 2003-February 2004 is approximately 0.042%, taking the yen from 110 per 1 dollar to 105 ($\simeq 110 \times (1 - 0.00042)^{111}$). According to estimation results, if there had been no intervention, $\hat{\alpha}_{0\cdot 1}$ would be 0.387% and 0.327% for the earlier and later periods, respectively. In other words, on days of intervention, the yen showed strong appreciation that was almost completely offset. It was not completely offset in the later period, but the yen's strengthening slowed considerably. Thus intervention was effective ($\hat{\alpha}_{\cdot 1} < 0$). Considering the standard deviation of volatility, it was $(\hat{\sigma}_{1\cdot 1},\ \hat{\sigma}_{0\cdot 1}) = (0.33,\ 0.43)$ in the earlier period and $(\hat{\sigma}_{1\cdot 1},\ \hat{\sigma}_{0\cdot 1}) = (0.45,\ 0.57)$ in the later period. It did decline due to intervention, but there is no significant difference from the t-value for VTE_1, indicating no accompanying reduction in volatility.

Furthermore, in the first structural equation—the authorities' response function—coefficients of determination are somewhat large. Thus, it can be deemed a response equation, which to some extent allows predicting an intervention from information up to the previous day. A noteworthy result is that β_1 does not differ significantly from zero for both earlier and later periods, indicating we cannot statistically infer that authori-

[2]Derivation of coefficients of determination is provided in the Appendix.

ties set intervention volume in response to currency movements at period t. Therefore, the simultaneous equation Tobit model can be deemed a triangular system[3]. Japanese authorities' response function was autonomous at that time, so it nearly equals the estimation results in Ito's (2003) response function. Considering the characteristics of the dynamic Tobit model, if the latent variables $y_t^{(1)*} = I_t^*$ for the previous and preceding days increase and authorities decide to intervene, steps will be taken to intervene irrespective of currency fluctuations during period t. This can somewhat be interpreted as continuous intervention, which is consistent with the intervention movements in Figure 2.

6 Conclusions

This study examined the efficacy of Japan's large-scale currency interventions between January 2003 and March 2004. Based on FIML estimator with ATE in a simultaneous equations Tobit model, we analyzed why interventions can be effective by considering their effects on exchange rates as their influence of an average trend. We considered these effects from two aspects based on structural estimation and used trend analysis for the estimation period based on a formulation of policy evaluation.

Our empirical results indicated that buying dollars equivalent to one trillion yen could induce a same-day depreciation exceeding 1% in the yen. These results suggested that intervention is an endogenous variable, however setting the volume of intervention is based on the variables of previous days. ATE_1 analysis revealed that intervention policy was almost completely offset by appreciation in the yen on the days of intervention. Because results indicated no influence of changes in exchange rates on volatility, we conclude that the primary objective of intervention was oriented toward the average trend. Consequently, the yen continued to appreciate during the period examined. Compared with a non-intervention policy, however, the pace of its increase slowed considerably. Thus, we conclude that Japan's intervention during the period examined was effective.

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Appendix

Derivation of likelihood function: We express the likelihood function equation (4.18) by using the cumulative distribution function of univariate normal distribution. The calculation for maximization is based on Ox (Doornik, 2002). The likelihood function for period t is as follows:

$$l_t = \left[\frac{(1-\beta_1\beta_2)}{(2\pi)^{\frac{2}{2}}|\boldsymbol{\Sigma}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}\mathbf{u}_{1t}'\boldsymbol{\Sigma}^{-1}\mathbf{u}_{1t}\right) \right]^{w_t} \left[\int_{-\infty}^{u_{1t}} g\left(u^{(1)},\, u_t^{(2)}\right) du^{(1)} \right]^{(1-w_t)} \tag{A.1}$$

where $\mathbf{u}_{1t} = (y_t^{(1)} - \beta_1 y_t^{(2)} - \boldsymbol{\gamma}_1'\mathbf{z}_t^{(1)},\, y_t^{(2)} - \beta_2 y_t^{(1)} - \boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)})'$ and $u_{1t} = -\beta_1 y_t^{(2)} - \boldsymbol{\gamma}_1'\mathbf{z}_t^{(1)}$. Consider the following orthogonal transformation of error terms:

$$y_t^{(1)*} = \beta_1 y_t^{(2)} + \boldsymbol{\gamma}_1'\mathbf{z}_{t-1}^{(1)} + \frac{\sigma_{12}}{\sigma_2^2}u_t^{(2)} + (u_t^{(1)} - \frac{\sigma_{12}}{\sigma_2^2}u_t^{(2)})\ , \tag{A.2}$$

where, for $u_t^{(1)*} = u_t^{(1)} - (\sigma_{12}/\sigma_2^2)u_t^{(2)}$, we have $\mathcal{C}ov[u_t^{(1)*},\ u_t^{(2)}] = 0$; therefore, $u_t^{(1)*}$ is independent of $u_t^{(2)}$. The Jacobian determinant of the variable transformation from $(u_t^{(1)}, u_t^{(2)})$ to $(u_t^{(1)*}, u_t^{(2)})$ is 1. Therefore, the second term of (A.1) can be rewritten as follows:

$$\begin{aligned}\int_{-\infty}^{u_{1t}-\frac{\sigma_{12}}{\sigma_2^2}u_t^{(2)}} g(u^{(1)*}, u_t^{(2)})du^{(1)*} &= \left(\int_{-\infty}^{(u_{1t}-\frac{\sigma_{12}}{\sigma_2^2}u_{2t})/\sigma_*} \phi(\frac{u^{(1)*}}{\sigma_*})\ d(\frac{u^{(1)*}}{\sigma_*}) \right) \frac{1}{\sigma_2}\phi\left(\frac{u_{2t}}{\sigma_2}\right) \\ &= \Phi\left(\frac{u_{1t} - \frac{\sigma_{12}}{\sigma_2^2}u_{2t}}{\sigma_*}\right) \times \frac{1}{\sqrt{2\pi}\sigma_2}\exp(-\frac{u_{2t}^2}{2\sigma_2^2})\ ,\end{aligned}$$

where $u_{2t} = y_t^{(2)} - \boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)}$ and $\sigma_*^2 = \sigma_1^2 - (\sigma_{12}^2/\sigma_2^2)$. From the above, if $(1-\beta_1\beta_2) > 0$, the log-likelihood function $\log L = \sum_t \log l_t$ is as follows:

$$\begin{aligned}\log L &= \sum_{t=1}^{T} w_t \left[\log(1-\beta_1\beta_2) - \log 2\pi - \log|\boldsymbol{\Sigma}|^{\frac{1}{2}} - \frac{1}{2}\mathbf{u}_{1t}'\boldsymbol{\Sigma}^{-1}\mathbf{u}_{1t}\right] \\ &+ (1-w_t)\left[\log\Phi\left(\frac{u_{1t} - \frac{\sigma_{12}}{\sigma_2^2}u_{2t}}{\sigma_*}\right) - \log\sqrt{2\pi} - \log\sigma_2 - \frac{u_{2t}^2}{2\sigma_2^2}\right] . \quad \text{(A.3)}\end{aligned}$$

Derivations of statistics for endogeneity: We derive equations (4.19) and (4.20). The structural equations are expressed as follows:

$$y_t^{(1)*} = \beta_1 y_t^{(2)} + \boldsymbol{\gamma}_1'\mathbf{z}_t^{(1)} + u_t^{(1)} = \boldsymbol{\theta}_1'\mathbf{x}_t^{(1)} + u_t^{(1)} , \quad \text{(A.4)}$$

$$y_t^{(2)} = \beta_2 y_t^{(1)} + \boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + u_t^{(2)} = \boldsymbol{\theta}_2'\mathbf{x}_t^{(2)} + u_t^{(2)} . \quad \text{(A.5)}$$

First, we consider the test statistic (4.20) of the second structural equation. Let the residuals be $\hat{u}_t^{(2)} = y_t^{(2)} - \hat{\boldsymbol{\theta}}_2'\mathbf{x}_t^{(2)}$, where $\hat{\boldsymbol{\theta}}_2$ is the FIML estimator for $(\beta_2, \boldsymbol{\gamma}_2')'$.

$$\frac{1}{\sqrt{T}}\sum_t y_t^{(1)}\hat{u}_t^{(2)} = \frac{1}{\sqrt{T}}\sum_t y_t^{(1)}u_t^{(2)} - \left[\frac{1}{T}\sum_t y_t^{(1)}\mathbf{x}_t^{(2)'}\right]\sqrt{T}(\hat{\boldsymbol{\theta}}_2 - \boldsymbol{\theta}_2) . \quad \text{(A.6)}$$

Further, consider the asymptotic linear approximation of $\sqrt{T}(\hat{\boldsymbol{\theta}}_2 - \boldsymbol{\theta}_2)$. We represent the log-likelihood function as $l(\boldsymbol{\theta}) = \sum_t l_t(\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2', \sigma_1, \sigma_2, \sigma_{12})'$ is a $((p_1 + p_2 + 3) \times 1)$ vector. If $\mathbf{s}_t = \mathbf{s}_t(\boldsymbol{\theta}) = \partial l_t(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ is the score function, from the discussions of the maximum likelihood method, we have

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = -\mathbf{H}^{-1}\left[\frac{1}{\sqrt{T}}\sum_t \mathbf{s}_t\right] + o_p(1) , \quad \text{(A.7)}$$

where $\mathbf{H} = (1/T)\sum_t \mathbf{H}_t(\boldsymbol{\theta})$ is the Hessian matrix. If the selection matrix $\mathbf{J}_2$ is $\mathbf{J}_2\boldsymbol{\theta} = \boldsymbol{\theta}_2$, then since $\sqrt{T}(\hat{\boldsymbol{\theta}}_2 - \boldsymbol{\theta}_2) = \sqrt{T}\mathbf{J}_2(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})$, (A.6) becomes

$$\frac{1}{\sqrt{T}}\sum_t y_t^{(1)}u_t^{(2)} + \mathbf{m}_{2T}'\left[\frac{1}{\sqrt{T}}\sum_t \mathbf{s}_t\right] + o_p(1) , \quad \text{(A.8)}$$

where it holds that $\mathbf{m}_{2T}' = \mathbf{m}_{2T}'(\boldsymbol{\theta}) = [(1/T)\sum_t y_t^{(1)}\mathbf{x}_t^{(2)'}]\mathbf{J}_2\mathbf{H}^{-1}$. Denoting its probability limit as $\mathbf{m}_2$, under the null hypothesis, the following holds:

$$\begin{aligned}\frac{1}{\sqrt{T}}\sum_t y_t^{(1)}\hat{u}_t^{(2)} &= \frac{1}{\sqrt{T}}\sum_t (y_t^{(1)}u_t^{(2)} + \mathbf{m}_2'\mathbf{s}_t) + o_p(1) \quad \text{(A.9)} \\ &\xrightarrow{d} \mathcal{N}(0, \sigma_{(2)}^2) , \quad \text{(A.10)}\end{aligned}$$

where we have $\sigma^2_{(2)} = \mathcal{E}[(y_t^{(1)}u_t^{(2)} + \mathbf{m}_2'\mathbf{s}_t)^2]$. Considering this consistent estimator $\hat{\sigma}^2_{(2)} = (1/T)\sum_t(y_t^{(1)}\hat{u}_t^{(2)} + \mathbf{m}_{2T}'(\hat{\boldsymbol{\theta}})\mathbf{s}_t(\hat{\boldsymbol{\theta}}))^2$ and normalizing yields equation (4.20).

Next, we consider the test statistic (4.19) for the first structural equation. From structural equation (4.14), we obtain $\mathcal{E}[y_t^{(2)}u_t^{(1)}] = \beta_2\mathcal{E}[y_t^{(1)}u_t^{(1)}] + \sigma_{12}$. Noting that $y_t^{(1)}u_t^{(1)} = y_t^{(1)}(y_t^{(1)} - \boldsymbol{\theta}_1'\mathbf{x}_t^{(1)})w_t$, the residuals are expressed as $\hat{u}_t^{(1)} = (y_t^{(1)} - \hat{\boldsymbol{\theta}}_1'\mathbf{x}_t^{(1)})w_t$. Thus, the following expression is obtained:

$$
\begin{aligned}
& \frac{1}{\sqrt{T}}\sum_t(\hat{\beta}_2 y_t^{(1)}\hat{u}_t^{(1)} + \hat{\sigma}_{12}) && \text{(A.11)} \\
= \ & \frac{1}{\sqrt{T}}\sum_t(\beta_2 y_t^{(1)}u_t^{(1)} + \sigma_{12}) - \frac{1}{T}\begin{bmatrix} -\sum_t -\beta_2 y_t^{(1)}\mathbf{x}_t^{(1)} \\ -\sum_t y_t^{(1)}u_t^{(1)} \\ -\sum_t 1 \end{bmatrix}' \begin{bmatrix} \sqrt{T}(\hat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1) \\ \sqrt{T}(\hat{\beta}_2 - \beta_2) \\ \sqrt{T}(\hat{\sigma}_{12} - \sigma_{12}) \end{bmatrix} \\
& -(\hat{\beta}_2 - \beta_2)\left[\frac{1}{T}\sum_t y_t^{(1)}\mathbf{x}^{(1)\prime}\right]\sqrt{T}(\hat{\boldsymbol{\theta}}_1 - \boldsymbol{\theta}_1) && \text{(A.12)} \\
= \ & \frac{1}{\sqrt{T}}\sum_t(\beta_2 y_t^{(1)}u_t^{(1)} + \sigma_{12}) - \boldsymbol{\mu}_{1T}'\mathbf{J}_1\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) + o_p(1)\ , && \text{(A.13)}
\end{aligned}
$$

where the third term of equation (A.12) is asymptotically negligible. Furthermore, $\boldsymbol{\mu}_{1T} = \boldsymbol{\mu}_{1T}(\beta_2, u_t^{(1)})$ is a $((p_1+1+1)\times 1)$ vector, and $\mathbf{J}_1$ is a selection matrix such that $(\boldsymbol{\theta}_1', \beta_2, \sigma_{12})' = \mathbf{J}_1\boldsymbol{\theta}$. Similar to equations (A.8) and (A.9), under the hypothesis $\mathrm{H}_0^{(1)}$, the following holds:

$$
\begin{aligned}
\frac{1}{\sqrt{T}}\sum_t(\hat{\beta}_2 y_t^{(1)}\hat{u}_t^{(1)} + \hat{\sigma}_{12}) &= \frac{1}{\sqrt{T}}\sum_t(\beta_2 y_t^{(1)}u_t^{(1)} + \sigma_{12} + \mathbf{m}_1'\mathbf{s}_t) + o_p(1) \\
&\xrightarrow{d} \mathcal{N}(0, \sigma^2_{(1)})\ , \qquad \text{(A.14)}
\end{aligned}
$$

where $\mathbf{m}_1'$ is the probability limit of $\mathbf{m}_{1T}'(\boldsymbol{\theta}) = \boldsymbol{\mu}_{1T}'\mathbf{J}_1\mathbf{H}^{-1}$, with $\sigma^2_{(1)} = \mathcal{E}[(\beta_2 y_t^{(1)}u_t^{(1)} + \sigma_{12} + \mathbf{m}_1'\mathbf{s}_t)^2]$. This consistent estimator is

$$
\hat{\sigma}^2_{(1)} = \frac{1}{T}\sum_t(\hat{\beta}_2 y_t^{(1)}\hat{u}_t^{(1)} + \hat{\sigma}_{12} + \mathbf{m}_{1T}'(\hat{\boldsymbol{\theta}})\mathbf{s}_t(\hat{\boldsymbol{\theta}}))^2\ , \qquad \text{(A.15)}
$$

where $\mathbf{m}_{1T}'(\hat{\boldsymbol{\theta}}) = \hat{\boldsymbol{\mu}}_{1T}'\mathbf{J}_1\mathbf{H}^{-1}$ and $\hat{\boldsymbol{\mu}}_{1T} = \boldsymbol{\mu}_{1T}(\hat{\beta}_2, \hat{u}_t^{(1)})$. By normalizing, we obtain equation (4.19).

Derivations of ATE$_1$ and VTE$_1$'s statistics: First, we consider equations (4.24) and (4.25). From the relationship with equation (4.17), $u_t^{(2)} = v_t^{(2)} - \beta_2 v_t^{(1)}$, we have

$$\begin{aligned} y_{1t}^{(2)} - y_{0t}^{(2)} &= \frac{\beta_2}{1-\beta_1\beta_2}\boldsymbol{\gamma}_1'\mathbf{z}_t^{(1)} + \frac{1}{1-\beta_1\beta_2}\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + v_t^{(2)} - (\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + u_t^{(2)}) \\ &= \frac{\beta_2}{1-\beta_1\beta_2}\boldsymbol{\gamma}_1'\mathbf{z}_t^{(1)} + \frac{\beta_1\beta_2}{1-\beta_1\beta_2}\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + \beta_2 v_t^{(1)} \\ &= \beta_2(\boldsymbol{\pi}_1'\mathbf{z}_t + v_t^{(1)}) \ . \end{aligned}$$

Substituting $y_t^{(2)} = 0 + \boldsymbol{\gamma}_2\mathbf{z}_t^{(2)} + u_t^{(2)}$ in equation (4.12) yields the reduced form $y_t^{(1)*} = (1-\beta_1\beta_2)(\boldsymbol{\pi}_1'\mathbf{z}_t + v_t^{(1)}) \leq 0$ when the latent variable $y_t^{(1)*}$ is negative. Hence,

$$\begin{aligned} \boldsymbol{\pi}_1'\mathbf{z}_t + u_t^{(1)} &= (\boldsymbol{\pi}_1'\mathbf{z}_t + u_t^{(1)})w_t + (\boldsymbol{\pi}_1'\mathbf{z}_t + u_t^{(1)})(1-w_t) \\ &= y_t^{(1)}w_t + \frac{1}{1-\beta_1\beta_2}y_t^{(1)*}(1-w_t) \ . \end{aligned}$$

Next, we consider the t-test of equation (4.26). Let a 3×1 vector be $\boldsymbol{\theta}_3 = (\beta_2, \mu_1, p_1)'$, where $\mu_1 = \mathcal{E}[y_t^{(1)}]$ and $p_1 = \mathcal{E}[w_t]$. From the consistent estimators $\hat{\mu}_1 = (1/T)\sum_t y_t^{(1)}$ and $\hat{p}_1 = (1/T)\sum_t w_t$ and the result of equation (A.9), we have $\sqrt{T}(\hat{\boldsymbol{\theta}}_3 - \boldsymbol{\theta}_3) \overset{d}{\to} \mathcal{N}(\mathbf{0}, \mathbf{V}_3)$. The consistent estimator of this asymptotic covariance matrix is given as follows:

$$\hat{\mathbf{V}}_3 = \frac{1}{T}\sum_t \mathbf{e}_{3t}\mathbf{e}_{3t}' \ , \ \ \mathbf{e}_{3t}' = (\hat{\epsilon}_t, \ y_t^{(1)} - \hat{\mu}_1, \ w_t - \hat{p}_1) \ , \tag{A.16}$$

where $\hat{\epsilon}_t = -\mathbf{e}_2'\mathbf{H}^{-1}\mathbf{s}_t$, and $\mathbf{e}_2$ is a selection vector set to $\beta_2 = \mathbf{e}_2'\boldsymbol{\theta}$. As $\hat{\alpha}_{\cdot 1} = (\hat{\beta}_2\hat{\mu}_1)/\hat{p}_1$, we evaluate it using the delta method. For $f(\boldsymbol{\theta}_3) = (\theta_1\theta_2)/\theta_3$, we have that $\mathbf{f}(\boldsymbol{\theta}_3) = \partial f(\boldsymbol{\theta}_3)/\partial\boldsymbol{\theta}_3 = (\theta_2/\theta_3, \theta_1/\theta_3, -(\theta_1\theta_2)/\theta_3^2)'$. Because we know $\mathbf{f}(\boldsymbol{\theta}_3) \neq \mathbf{0}$ from $\mu_1/p_1 \neq 0$, the distribution is not degenerate. Therefore the t-test statistic for the null hypothesis $\mathrm{H}_0 : f(\boldsymbol{\theta}_3) = 0$ is given by

$$t = \frac{\sqrt{T}f(\hat{\boldsymbol{\theta}}_3)}{\sqrt{\mathbf{f}'(\hat{\boldsymbol{\theta}}_3)\hat{\mathbf{V}}_3\mathbf{f}(\hat{\boldsymbol{\theta}}_3)}} \ . \tag{A.17}$$

Further, let us represent the α_{01} of equation (4.27). $\alpha_{0\cdot 1} = \mathcal{E}[\mathcal{E}[\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + u_t^{(2)}|\mathbf{z}_t, w_t = 1]|w_t = 1]$; therefore, considering the discussion in Maddala (1983, Appendix), we have

$$\begin{aligned} \mathcal{E}[\boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + u_t^{(2)}|\mathbf{z}_t, w_t = 1] &= \boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} - \frac{\mathcal{E}[v_t^{(1)}u_t^{(2)}]}{\omega_1^2}\omega_1\mathcal{E}\left[\frac{-v_t^{(1)}}{\omega_1}\Big|\frac{-v_t^{(1)}}{\omega_1} < \frac{\boldsymbol{\pi}_1'\mathbf{z}_t}{\omega_1}\right] \\ &= \boldsymbol{\gamma}_2'\mathbf{z}_t^{(2)} + \frac{\sigma_{12} + \beta_1\sigma_2^2}{(1-\beta_1\beta_2)\omega_1}\frac{\phi(\boldsymbol{\pi}_1'\mathbf{z}_t)}{\Phi(\boldsymbol{\pi}_1'\mathbf{z}_t)} \ , \end{aligned}$$

where $\omega_1^2 = \mathcal{V}ar[v_t^{(1)}]$, Φ and ϕ represent the standard normal cumulative distribution function and its density function, respectively, and we use the relations that $\mathcal{E}[u_t^{(2)}| - v_t^{(1)}] = -(\mathcal{E}[v_t^{(1)}u_t^{(2)}]/\omega_1^2)(-v_t^{(1)})$ and $\mathcal{E}[v_t^{(1)}u_t^{(2)}] = (\sigma_{12} + \beta_1\sigma_2^2)/(1-\beta_1\beta_2)$.

Finally, we derive the VTE$_1$ estimator of equation (4.31). From the relationships with $\mathcal{V}ar[y_{1t}^{(2)}|w_t = 1] = \mathcal{V}ar[y_t^{(2)}|w_t = 1]$ and $\mathcal{V}ar[y_t^{(2)}|w_t = 1] = \mathcal{E}[y_t^{(2)2}|w_t = 1] - (\mathcal{E}[y_t^{(2)}|w_t = 1])^2$, we obtain

$$\hat{\sigma}_{1\cdot 1}^2 = \frac{1}{\sum_t w_t} \sum_t y_t^{(2)2} w_t - (\hat{\alpha}_{1\cdot 1})^2 \ . \tag{A.18}$$

From equation (4.24), $\mathcal{V}ar[y_{0t}^{(2)}|w_t = 1] = \mathcal{V}ar[y_t^{(2)} - \beta_2 y_t^{(1)}|w_t = 1]$, so that

$$\hat{\sigma}_{0\cdot 1}^2 = \frac{1}{\sum_t w_t} \sum_t (y_t^{(2)} - \hat{\beta}_2 y_t^{(1)})^2 w_t - (\hat{\alpha}_{0\cdot 1})^2 \ . \tag{A.19}$$

Therefore, we obtain the VTE$_1$ consistent estimator $(\hat{\sigma}_{1\cdot 1}^2 - \hat{\sigma}_{0\cdot 1}^2)$. Next, we consider the t-value. VTE$_1$ is also expressed as follows:

$$\hat{\sigma}_{1\cdot 1}^2 - \hat{\sigma}_{0\cdot 1}^2 = 2\hat{\beta}_2 \left[\frac{\hat{m}_{12}}{\hat{p}_1} - \frac{\hat{\mu}_1}{\hat{p}_1} \frac{\hat{\mu}_2}{\hat{p}_1} \right] - \hat{\beta}_2^2 \left[\frac{\hat{m}_1^2}{\hat{p}_1} - \left(\frac{\hat{\mu}_1}{\hat{p}_1} \right)^2 \right] \ , \tag{A.20}$$

where $\hat{m}_1^2 = (1/T) \sum_t y_t^{(1)2}$, $\hat{m}_{12} = (1/T) \sum_t y_t^{(1)} y_t^{(2)}$, and $\hat{\mu}_2 = (1/T) \sum_t y_t^{(2)} w_t$. Hence, we evaluate using the delta method. We set up a 6×1 vector $\boldsymbol{\theta}_4 = (\beta_2, \mu_1, p_1, m_1^2, m_{12}, \mu_2)$, where $m_1^2 = \mathcal{E}[y_t^{(1)2}]$, $m_{12} = \mathcal{E}[y_t^{(1)} y_t^{(2)}]$, and $\mu_2 = \mathcal{E}[y_t^{(2)} w_t]$. For $g(\boldsymbol{\theta}_4) = 2\theta_1[\theta_5/\theta_3 - \theta_2\theta_6/\theta_3^2] - \theta_1^2[\theta_4/\theta_3 - (\theta_2/\theta_3)^2]$, $\mathbf{g}(\boldsymbol{\theta}_4) = \partial g(\boldsymbol{\theta}_4)/\partial \boldsymbol{\theta}_4$ becomes the following:

$$\mathbf{g}(\boldsymbol{\theta}_4) = \begin{bmatrix} 2(\theta_5 - \theta_1\theta_4)\theta_3^{-1} - 2(\theta_1\theta_2^2 - \theta_2\theta_6)\theta_3^{-2} \\ 2(\theta_1^2\theta_2 - \theta_1\theta_6)\theta_3^{-2} \\ (\theta_1^2\theta_4 - 2\theta_1\theta_5)\theta_3^{-2} - 2(\theta_1^2\theta_2^2 - 2\theta_1\theta_2\theta_6)\theta_3^{-3} \\ -\theta_1^2\theta_3^{-1} \\ 2\theta_1\theta_3^{-1} \\ -2\theta_1\theta_2\theta_3^{-2} \end{bmatrix} \ . \tag{A.21}$$

The estimator corresponding to equation (A.16) is $\hat{\mathbf{V}}_4 = (1/T) \sum_t \mathbf{e}_{4t}\mathbf{e}_{4t}'$, where $\mathbf{e}_{4t}' = (\mathbf{e}_{3t}',\ y_t^{(1)2} - \hat{m}_1^2,\ y_t^{(1)} y_t^{(2)} - \hat{m}_{12},\ y_t^{(2)} w_t - \hat{\mu}_2)$. Under a sufficient condition $\beta_2 \neq 0$ for $\mathbf{g}(\boldsymbol{\theta}_4) \neq 0$, the t-test statstic is given by $t = \sqrt{T} g(\hat{\boldsymbol{\theta}}_4)/[\mathbf{g}'(\hat{\boldsymbol{\theta}}_4)\hat{\mathbf{V}}_4 \mathbf{g}(\hat{\boldsymbol{\theta}}_4)]^{1/2}$.

Derivations of the test statistic for structural changes: The sample used herein comprises daily data from January 1, 2003 to April 28, 2004. As the G7 meeting was scheduled for September 20, 2003, tripartite meetings of officials from Japan, Germany, and the US were held through the preceding day. In this meeting, negative opinions were expressed concerning Japanese frequent interventions, and as shown in Figure 2, the market response comprised a sharp appreciation of the yen. Thus, there were strong exogenous factors for exchange rate fluctuations from September 19 to September 22, 2003. Therefore, we excluded this period from the analysis and there may have been structural changes before and after this period.

We denote the structural parameters $(\beta_1, \beta_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\Sigma})$ for the previous period as a $K \times 1$ vector $\boldsymbol{\theta}_{03}$ and for the succeeding period, similarly, as $\boldsymbol{\theta}_{04}$. We also express the null hypothesis $\mathrm{H}_0 : \boldsymbol{\theta}_{03} = \boldsymbol{\theta}_{04}$ as $\mathbf{R}(\boldsymbol{\theta}'_{03}, \boldsymbol{\theta}'_{04})' = \mathbf{0}$, where $\mathbf{R} = [\mathbf{I}_K, -\mathbf{I}_K]$. Let the estimators for the asymptotic covariance matrix of the FIML estimators $(\hat{\boldsymbol{\theta}}_{03}, \hat{\boldsymbol{\theta}}_{04})$ for the previous and succeeding periods be $(\hat{\mathbf{V}}_{03}, \hat{\mathbf{V}}_{04})$. Then, the Wald-type test statistic is given by

$$q = \mathbf{e}'(\mathbf{R}\hat{\mathbf{V}}\mathbf{R}')^{-1}\mathbf{e} \ \xrightarrow{d} \ \chi^2(K) \ , \tag{A.22}$$

where T_1 and T_2 are the sample size for the previous and succeeding periods, respectively, then it follows that

$$\mathbf{e} = \mathbf{R}\begin{bmatrix} \sqrt{T_1}\hat{\boldsymbol{\theta}}_{03} \\ \sqrt{T_2}\hat{\boldsymbol{\theta}}_{04} \end{bmatrix} , \qquad \hat{\mathbf{V}} = \begin{bmatrix} \hat{\mathbf{V}}_{03} & \mathbf{O} \\ \mathbf{O} & \hat{\mathbf{V}}_{04} \end{bmatrix} . \tag{A.23}$$

Note that $\hat{\mathbf{V}}$ is the block diagonal matrix, since $\mathcal{E}[\partial \log L / \partial \boldsymbol{\theta}_{03} \partial \boldsymbol{\theta}'_{04}] = \mathbf{O}$.

Derivations of Coefficients of determination: There are many approaches to Tobit model's coefficient of determination, as described in the survey article by Veall and Zimmerman (1996). For the two reduced forms (4.15) and (4.16) that examined herein, we consider the following variance ratios:

$$0 \leq \frac{\mathcal{V}ar[\ \mathcal{E}[y_t^{(g)}|\mathbf{z}_t]\ ]}{\mathcal{V}ar[y_t^{(g)}]} = 1 - \frac{\mathcal{V}ar[\epsilon_t^{(g)}]}{\mathcal{V}ar[y_t^{(g)}]} \leq 1 \ , \qquad g = 1,\ 2 \ . \tag{A.24}$$

The advantage of this approach is that the conditional expectation in the numerator and $\epsilon_t^{(g)} = y_t^{(g)} - \mathcal{E}[y_t^{(g)}|\mathbf{z}_t]$ is uncorrelated, meaning that they can be interpreted in the same way as a usual linear regression analysis. The conditional expectations including $y_t^{(1)} = 0$ with $\mathbf{z}_t$ as given are as follows:

$$\mathcal{E}[y_t^{(1)}|\mathbf{z}_t] \ = \ (\boldsymbol{\pi}'_1\mathbf{z}_t)\Phi(\frac{\boldsymbol{\pi}'_1\mathbf{z}_t}{\omega_1}) + \omega_1\phi(\frac{\boldsymbol{\pi}'_1\mathbf{z}_t}{\omega_1}) \geq 0 \ \ a.s. \ , \tag{A.25}$$

$$\mathcal{E}[y_t^{(2)}|\mathbf{z}_t] \ = \ \beta_2\mathcal{E}[y_t^{(1)}|\mathbf{z}_t] + \boldsymbol{\gamma}'_2\mathbf{z}_t^{(2)} \ . \tag{A.26}$$

The reduced form parameters in equations (A.25) and (A.26) can be estimated from the relationship between (4.17). We denote these predicted values as $\hat{y}_t^{(g)}$ $(g = 1,\ 2)$, then the coefficients of determination $R^2_{(g)}$ are

$$R^2_{(g)} \ = \ \frac{\sum_{t=1}^{T}(\hat{y}_t^{(g)} - \bar{\hat{y}}_t^{(g)})^2}{\sum_{t=1}^{T}(y_t^{(g)} - \bar{y}_t^{(g)})^2} \ , \qquad g = 1,\ 2 \ , \tag{A.27}$$

where $\bar{y}_t^{(g)}$ and $\bar{\hat{y}}_t^{(g)}$ are the sample averages for $y_t^{(g)}$ and $\hat{y}_t^{(g)}$, respectively. $\mathbf{z}_t$ is composed almost entirely of predetermined variables; therefore, its meaning covers the extent to which the intervention volume and change in exchange rate can be predicted using information of up till the preceding day.